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Vertical Momentum of the Fountain Produced by Multijet Vertical Impingement on a Flat Ground Plane

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An analysis of the vertical momentum flux for the fountain produced by multijet vertical impingement on a flat ground plane is presented. The jets are considered to have equal thrust and the same exit diameter, equally spaced on a bolt circle. Analytical formulas for both the core and arms of the fountain have been derived and show the dependence on the height, planform size, and number of jets. Preliminary numerical comparison with some experimental data indicates the formulas to be capable of producing acceptable predictions. Comparison made with Kuhn's empirical formulas revealed some similarity as well as differences between the two sets of formulas. Additional work needed for the development of prediction methods for lift losses is suggested.

Nomenclature

d	= jet exit diameter
d_e	= diameter of the equal single jet
$\bar{D}$	= equivalent planform diameter
D'	= diameter of the bolt line circle
e	= distance between adjacent jet centers
h	= planform height
h'	= height at which the core lift increment vanishes
L	= length of a finite plane
ΔL_2	= lift increment of the fountain core
ΔL_3	= lift increment of the fountain arms
m	= index for the fountain width, Eq. (8)
M_y	= vertical momentum flux (y direction)
$M_{y,L}$	= vertical momentum flux of a finite plane
$M_{y,a}$	= vertical momentum flux of the fountain arms
$M_{y,c}$	= vertical momentum flux of the fountain core
n	= index in the fountain velocity, Eq. (7)
N	= number of jets
O	= origin of the coordinates
O^*	= origin with respect to the projected jet center
P	= field point
r	= distance between O^* and P
s	= shape factor
T	= total thrust of jets
V	= velocity of the fountain at the plane of symmetry
w	= fountain width
x,y,z	= Cartesian coordinate (Fig. 1)
θ	= angle (Fig. 1)
Θ	= solidity factor of jet curtain
λ	= index of height decay, Eq. (9)
ρ	= density

Introduction

IN the design of V/STOL aircraft, lift losses in the hovering-flight mode of the aircraft present a major problem. At present, a reliable prediction method is still not available, although considerable analytical and experimental efforts have been expended on the problem for many years (e.g., Refs. 1-6). The method devised by Karemaa et al.[2,4] is basically empirical and requires, for its implementation, an adequate data base not yet in existence. Currently, this method is being further developed with additional experimental and analytical work at General Dynamics, Forth Worth (e.g., Ref. 4).

Recently, in a technical report prepared for the Naval Air Development Center, Kuhn[7] presented a method for estimating the lift losses experienced by a multijet V/STOL aircraft hovering in ground effect. Many useful formulas for the suckdown and fountain lift increment have been obtained. Kuhn's method was developed by correlating available data on flat plate and low wing configurations; consequently, it has a limited scope of applicability.

Kuhn's method and formulas, although empirical in nature, do provide some helpful insight of the problem. For example, his formula for the lift increment due to fountain core is shown to be a function of the "solidity of the jet curtain." It appears that Kuhn's method does have physical bases; however, his approach is empirical.

In the present study, an attempt is made to develop an analytical approach for the aerodynamics of lift losses and derive analytical formulas for lift losses by making use of some simplifying assumptions and hypotheses. As will be discussed later in this report, these assumptions or hypotheses are basic in nature and have some experimental evidence of validity. The work reported here will be concerned with the lift increment produced by multijet impingement in ground effect. The other key problem in lift losses, i.e., the suckdown, will be considered later.

Two-Jet Case: Some Basic Considerations

Some basic aerodynamic features produced by two interacting, initially axisymmetric jets impinging perpendicularly on a flat ground plane are shown in Fig. 1. The two jets whose exit planes are at the same height above the ground have equal thrust and the same exit diameter d. The centerlines of the two jets are separated by a distance e. Figure 2 shows the coordinates used in the analysis and the merging of the ground streamlines to form the midplane of the fountain. The total momentum flux in the vertical direction y across a plane perpendicular to y is

$$M_y = \rho \int_{-\infty}^{\infty} sw(V\cos\theta)^2 \mathrm{d}x^* \tag{1}$$

where w is the width of the fountain, V the velocity of fountain flow in x-y plane, and s the shape factor.

The assumption is made that V is in the radial direction r with its origin at the point O^*, i.e., the projected jet center (Fig. 2). This assumption has some experimental support (see, e.g., Ref. 5). In addition, if the momentum flux is assumed to be conserved for both the wall jets and the fountain, then

$$\rho sw V^2 r = C = \text{const} \tag{2}$$

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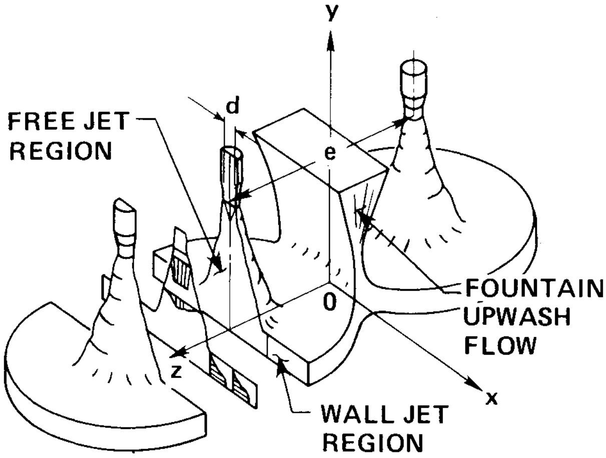

Fig. 1 Two interacting, initially axisymmetrical jets impinging perpendicularly on the ground with fountain formation.

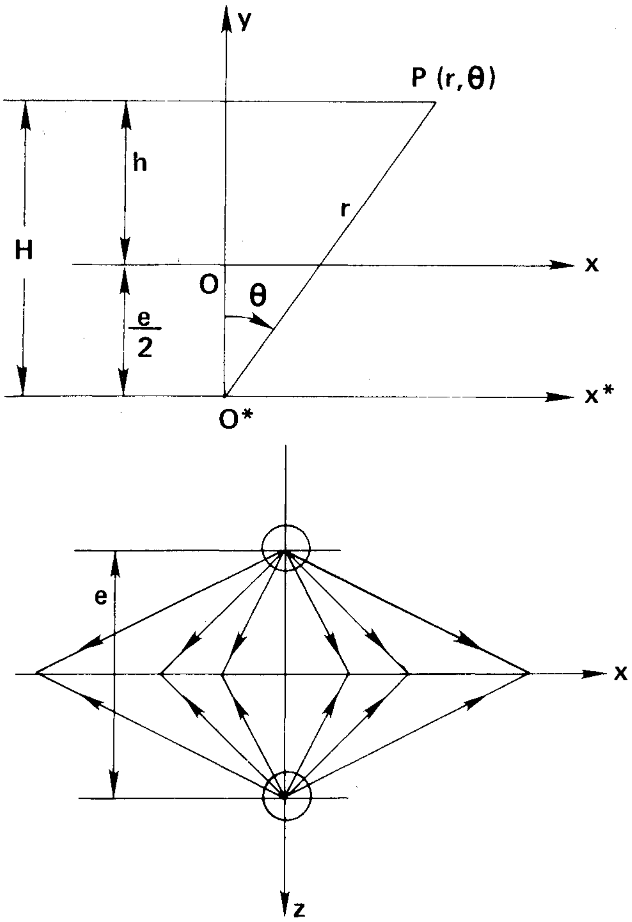

Fig. 2 Two-jet impingement problem.

Since $x^* = r\sin\theta$ and $H = r\cos\theta$, it is found

$$dx^* = H d\theta/\cos^2\theta$$

Thus, from Eqs. (1) and (2)

$$M_y = C\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = 2C \tag{3}$$

Now C can be expressed in terms of the total thrust of the two jets

$$C = T/2\pi \tag{4}$$

Thus, the vertical momentum flux M_y is

$$M_y = T/\pi \tag{5}$$

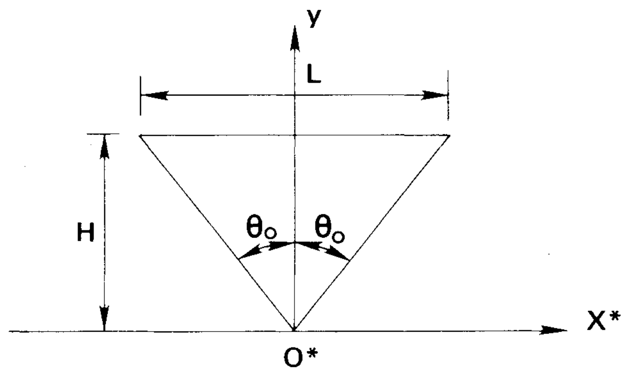

Fig. 3 Finite plane of length L.

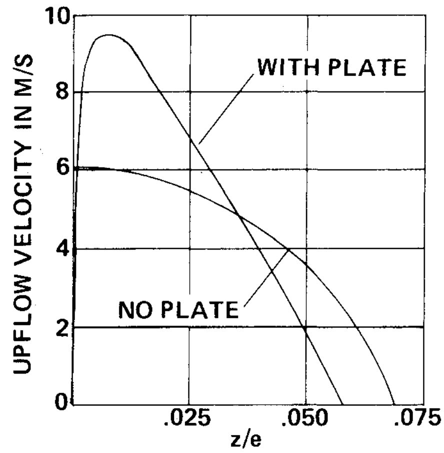

Fig. 4 Effect of a reflection plate on the upflow; two equal jets.

This expression of M_y shows no dependence on h, the height from the ground.

Next, consider a finite plane of length L (Fig. 3). The momentum flux $M_{y,L}$ is

$$M_{y,L} = \frac{T}{\pi}\sin\theta_0 = \frac{T}{\pi}\frac{L}{[L^2 + (e+2h)^2]^{1/2}} \tag{6}$$

Both expressions Eqs. (5) and (6) are expected to have limited validity. It is necessary to re-examine the assumptions used in the derivation.

Hertel[1] has carried out experiments to measure the velocity of the upflow in the fountain and found that if a reflection plate was inserted at the plane of symmetry, there occurred a dramatic change in the velocity field as shown in Fig. 4. In the presence of the plate, the upflow velocity assumes a form characteristic of a wall jet. However, the velocity field in the fountain is evidently different from that of a wall jet.

In addition, measurements by Grumman[3] for the upwash total pressure show its decay with distance to be different from that of the wall jet. A typical example is shown in Fig. 5. Using the jet center as the virtual origin, the decay is approximately $r^{-3.8}$, considerably more rapid than the decay of the square of the maximum velocity of a wall jet.

In Fig. 6, two regions are identified—the lower wall jet and the upper fountain. Consider a point $P_e(r_e,\theta)$ at the boundary of these two regions. The maximum velocity and width of the fountain at P are V_e and w_e, respectively. The maximum velocity and fountain width at a point P are assumed to be

$$(V/V_e)^2 = (r_e/r)^n \tag{7}$$

$$w/w_e = (r/r_e)^m \tag{8}$$

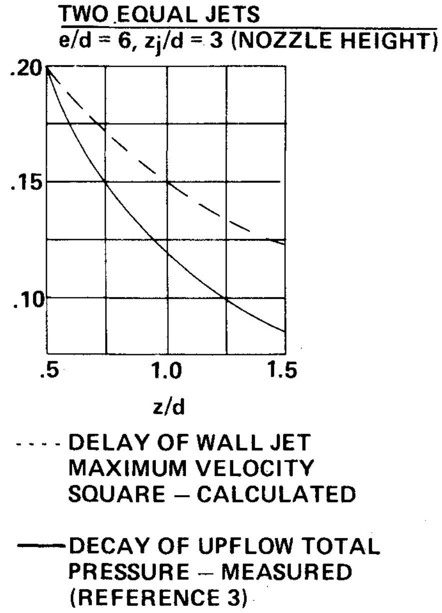

Fig. 5 Comparison of decay rates.

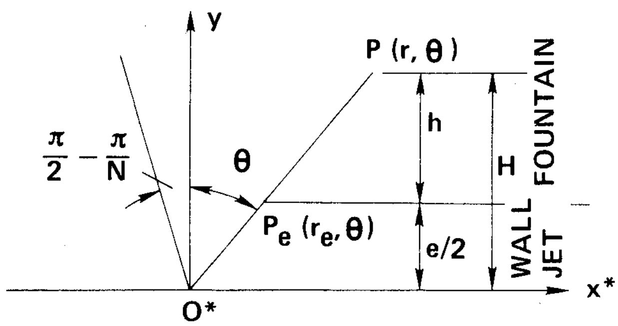

Fig. 6 Flow regions and coordinates.

The preceding representation for the velocity and width of the fountain is chosen for convenience. It is possible, upon further study, that V and w may be found to depend on both r and θ.

Substituting the preceding relations into Eq. (1) instead of Eq. (5) yields the following expression for M_y:

$$M_y = \frac{1}{[1+(2h/e)]^{n-1-m}} \frac{T}{\pi} \tag{9}$$

In obtaining Eq. (9), the geometric relations $H = r\cos\theta$, $e = 2r_e \cos\theta$, and $H = h + e/2$ are used (see Fig. 6). In addition, it is assumed that

$$\rho s_e w_e V_e^2 r_e = C = \text{const}$$

i.e., there is no momentum loss in the wall jet, an acceptable assumption suggested by the measurements of Donaldson and Snedeker.[8] Equation (9) shows that the decrement in M_y is due to momentum loss in the fountain. Evidently, the parameter $\lambda = n - 1 - m$ vanishes when the momentum in the fountain is conserved. As indicated already, the value of n has been estimated to be about 3.8 based on Grumman measurements. An accurate value for m is not available, but Grumman's measurements suggest that m is not high enough to make λ vanish. A reasonable value for m appears to be about 1.5. Thus, it is suggested for convenience to take λ to be 1.

According to a personal communication, Dr. D. Kotansky[9] from his measurements at MCAIR found strong evidence of momentum loss, which amounts to, for example, 50% of the incoming momentum flux in the x-y plane at a height of $y = d$ in the fountain of a two-jet case. However, the mechanism responsible for the loss has not been identified precisely.

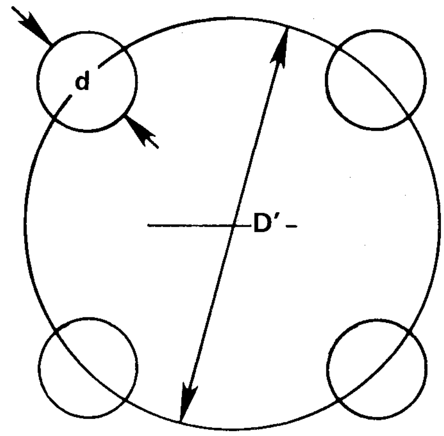

Fig. 7 Four equal jets; D'-bolt circle diameter.

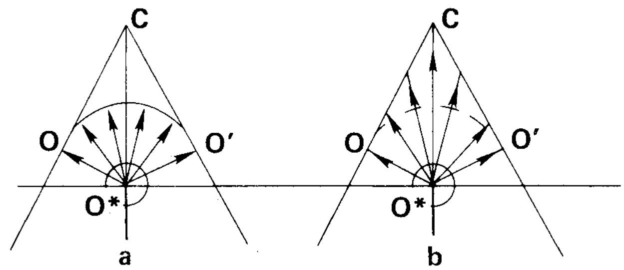

Fig. 8 Two models for fountain core.

Multijet Vertical Impingement

Figure 7 shows a four-jet arrangement with D' as the bolt circle diameter. It is well known that when the number of jets is greater than two, the fountain will be made of a core and several arms (four arms for the four-jet arrangement). Formulas for the vertical momentum flux in the fountain will be derived in the following analysis. A basic assumption is that all the streamlines are straight lines, and the streamline initiating from O^* and reaching the line OC perpendicularly will turn up 90 deg at O and become vertical in the direction of y axis (Fig. 8). The vertical portion of this streamline is the boundary between the core and arm of the fountain in the midplane of two adjacent jets. All the multijet considered in the present work have the jets equally spaced around the bolt circle, with the jets having the same thrust and equal exit diameter d.

Core

At present the knowledge of the fountain is not sufficient to allow an analysis without introducing additional assumption for the fountain flow. In Fig. 8 a segment of the flow in the ground plane for N jets is shown and two flow patterns are sketched. In Fig. 8a the portion of the wall jet segment is assumed to turn up vertically at the arc OO' to form the fountain core. On the other hand, as shown in Fig. 8b, the wall jet is assumed to turn up along OC and $O'C$.

The momentum flux for a segment of the fountain core for N jets (Fig. 8) is equal to $T/2\pi N$, where T is the total thrust of the jets. If the model shown in Fig. 8a is used for the fountain, the total vertical momentum flux of the core is equal to

$$(M_{y,c})_a = 2N\int_0^{\pi/2-\pi/N} \frac{T}{2\pi N} \mathrm{d}\theta = T\left(\frac{1}{2} - \frac{1}{N}\right) \tag{10}$$

Table 1 Numerical values for the momentum flux

N	$(M_{y,c})_a$	$(M_{y,c})_b$
2	0	0
3	1/6	$\frac{1}{2\pi}$
4	1/4	$\frac{1}{\sqrt{2}\pi}$
6	1/3	$\frac{\sqrt{3}}{2\pi}$
∞	1/2	$\frac{1}{\pi}$

If the model in Fig. 8b is used,

$$(M_{y,c})_b = \frac{T}{\pi}\int_0^{\pi/2-\pi/N}\cos\theta\,d\theta = \frac{T}{\pi}\cos\frac{\pi}{N} \qquad (11)$$

Equations (10) and (11) are not expected to be very accurate, since the models shown in Figs. 8a and 8b are highly idealized. Evidently $(M_{y,c})_b \leq (M_{y,c})_a$ as shown in Table 1. It is possible that $(M_{y,c})_a$ is more accurate for larger values of N, while $(M_{y,c})_b$ is more accurate for $N=3$. Future measurements may confirm the expectation that $(M_{y,c})_a$ and $(M_{y,c})_b$ are the upper and lower bounds of the core momentum flux.

The same expression $(M_{y,c})_a$ has been obtained by Kotansky and Glaze,[6] who also found out the actual values from their measurements to be less by a factor of ⅓ to ¼.

Using the same approximation for the momentum loss in the fountain as in the two-jet case, one can obtain the following expression for $M_{y,c}$ which will be used for further study in this work:

$$M_{y,c} = \frac{T}{\pi(1+2h/e)}\cos\frac{\pi}{N} \qquad (12)$$

The preceding expression vanishes when $N=2$. Note that the distance e between the centerlines of two adjacent jets is related to the diameter D' of the bolt circle and N by the equation $e=D'\sin(\pi/N)$ (see Fig. 8). Thus, Eq. (12) can be written in the following form:

$$M_{y,c} = \frac{T}{2\pi}\frac{D'}{D'\sin(\pi/N)+2h}\sin\frac{2\pi}{N} \qquad (13)$$

Using the preceding formula, we can show that for given values of T and D', the number of jets that maximizes the vertical momentum flux $M_{y,c}$ varies with the planform height h. For example, for values of $h/D'=0.125$ and 0.3285, the desired number of jets is $N=6$ and 5, respectively. However, these values of N are valid for $\lambda=1$. If the value of λ is taken to be 2, the values of N become 5 and 4 approximately.

Fountain Arms

Based on the preceding approximations, the following formula for the vertical momentum flux for N jets is obtained for arms extending to a radial distance $\bar{D}/2$ from the fountain axis. In Kuhn's study[7] $\bar{D}$ may be regarded as equivalent to the mean "planform diameter."

$$M_{y,a} = \frac{T}{\pi}\frac{\bar{D}}{[1+(2h/e)][\bar{D}^2+(2h+e)^2]^{1/2}} \qquad (14)$$

In contrast to the momentum flux of the core, the momentum flux $M_{y,a}$ for the arms is found to vary little with N, when N is changed from 2 to 6, for example.

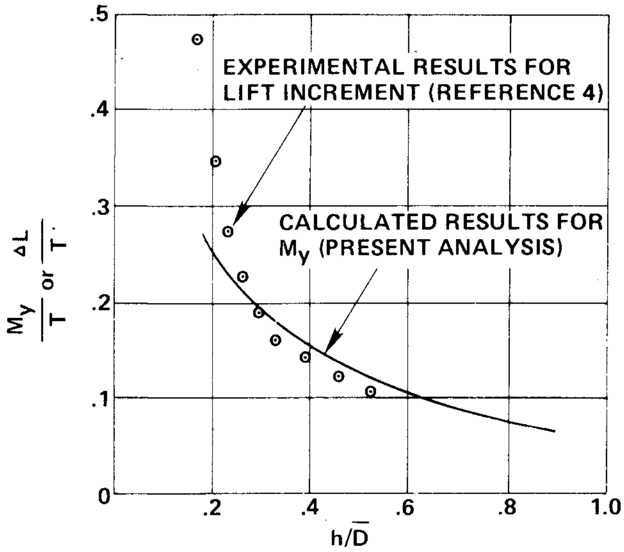

Fig. 9 Lift increment and vertical momentum flux M_y; four jets, $D'/\bar{D}=0.7$.

The dimensionless parameters for the N jet configuration are

$$\begin{aligned} N &= \text{number of jets} \\ D'/\bar{D} &= \text{planform size} \qquad (15) \\ h/\bar{D} &= \text{height} \end{aligned}$$

It might be expected that the ratio of the distance (e) between two adjacent jets and the jet exit diameter (d) should appear as a separate parameter in the preceding expressions for the momentum flux. The fact that the ratio did not appear suggests that validity of the expressions is limited to values of e/d within a certain range. The lower limit is likely to be 3, but the upper limit remains to be determined. No actual planform is assumed to be present in the analysis. Otherwise, suckdown effect has to be considered.

The numerical results for the total momentum flux $M_y = M_{y,c}+M_{y,a}$ for the case whose lift increments have been reported in Ref. 4 is shown in Fig. 9. The case is a four-jet configuration with $D'/\bar{D}=0.7$. Figure 9 shows the comparison between the calculated results for M_y [based on Eqs. (12) and (14)] and the experimental results from Ref. 4. The agreement between the calculated and experimental results cannot be considered as entirely satisfactory, but appears to be adequate to support the validity of the method of approach used in the present analysis.

A comparison of the formulas for lift increments, Eqs. (12) and (14), with those obtained by Kuhn[7] is given in the Appendix. Kuhn also showed other experimental results including some cases for which the lift increment vanishes at h'. The case chosen for comparison for the present analysis does not exhibit this behavior.

Concluding Remarks

An analysis of the vertical momentum flux for the fountain produced by multijet vertical impingement on a flat ground plane is presented. Analytical formulas for the core and arms of the fountain have been derived. An evaluation of these formulas for applicability to the problem of lift loss is being conducted by R.E. Kuhn under a contract to the U.S. Navy. It is assumed in this evaluation that the momentum loss does occur, although, as mentioned already, the mechanism remains to be investigated.

If a finite cylindrical volume in the fountain is considered, there will be momentum loss produced by turbulent mixing.

To show such a momentum loss, consider a round turbulent jet issuing from the origin O (Ref. 10). The vertical momentum flux at a height h from the origin over a circle with diameter $\bar{D}$ can be evaluated by using the analytical expression for the vertical velocity given in Ref. 10, and is found to be

$$\left(\frac{k\bar{D}}{h}\right)^2 \frac{3+3(k\bar{D}/h)^2+(k\bar{D}/h)^4}{1+3(k\bar{D}/h)^2+3(k\bar{D}/h)^4+(k\bar{D}/h)^6}$$

where k is a constant (equal to approximately 3.8). Thus, for small values of $\bar{D}$ the momentum loss is proportional to h^{-2} (for sufficiently large values of h). Only when $\bar{D}$ becomes infinitely large or h is equal to zero will the momentum loss vanish. An experimental study is being initiated to investigate the momentum loss in the fountain produced by multijet impingement on a flat ground plane.

Appendix: Comparison with Kuhn's Work

Kuhn's formulas for the lift increment are as follows:

Core

$$\frac{\Delta L_2}{T} = 0.135 N^2 \Theta \left(\frac{h'}{\bar{D}} - \frac{h}{\bar{D}}\right) \tag{A1}$$

$$\frac{h'}{\bar{D}} = 1.22 \frac{1}{N} \left(\frac{D'}{d_e}\right)^{0.65} \tag{A2}$$

Arms

$$\frac{\Delta L_3}{T} = 10^{(0.9 - 10h/\bar{D})} \tag{A3}$$

The core lift increment is assumed to be zero when $N=2$. The expression Eq. (12), of course, vanishes automatically at $N=2$. The quantity h' is the height at which the core lift increment becomes zero. Since the jet merging is not included in the present analysis, prediction of this height is beyond its scope. In Eq. (A2) d_e is the diameter of "equivalent single jet" and is equal to $N^{1/2} d$ for circular jet exit with diameter d. The fraction of jet pattern circumference Θ blocked by individual jets can be taken as $\pi D'/N$. Thus Eq. (A1) can be written as

$$\frac{\Delta L_2}{T} = 0.135 \pi D' N \frac{h' - h}{\bar{D}} \tag{A4}$$

Due to the difference in approaches, it is difficult to make analytical comparison of Eqs. (12) or (13) and (14) with Kuhn's (A1) and (A3). There is some similarity between Eqs. (13) and (A4) for the core. For the arms, while Eq. (14) is dependent on the parameters N, $d'/\bar{D}$, and $h/\bar{D}$, Kuhn's expression Eq. (A3) is a function of $h/\bar{D}$ only. The numerical values obtained from Eq. (14) are generally much higher than those from Eq. (A3). It is of interest to note that except $h'/\bar{D}$ Kuhn's formulas do not have the jet exit diameter d as a parameter.

References

[1] Hertel, H., "Wandstromungen und Aufstrome aus der Umlenkung von Freistrahlgruppen" (Wall Flows and Up-Streams Due to the Diversion of Free Jet Groups), *Fortschritt Berichce VDI Zeitschrift Fortschr.*, Vol. 12, July 1966, pp. 1-72.

[2] Karemaa, A. and Ramsey, J.C., "Aerodynamic Methodology for the Prediction of Jet-Induced Lift in Hover," General Dynamics, Convair Aerospace Division, Rept. CASD-ERR-73-012, Dec. 1973.

[3] Siclari, M.J., Hill, W.G., and Jenkins, R.C., "Investigation of Stagnation Line and Upwash Formation," AIAA Paper 77-615, 1977.

[4] Karemaa, A., Smith, C.W., Weber, H.A., and Garner, J.E., "The Aerodynamic and Thermodynamic Characteristics of Fountains and Some Far-Field Temperature Distributions," Office of Naval Research, Rept. under Contract N00014-76-C-0698, May 1978.

[5] Kotansky, D.R., Durando, N.A., Bristow, D.R., and Saunders, P.W., "Multi-Jet Induced Forces and Moments on VSTOL Aircraft Hovering In and Out of Ground Effect," McDonnell Aircraft Company, NADC-77-229-30, 1977.

[6] Kotansky, D.R. and Glaze, L.W., "Development of an Empirical Data Base and Analytical Modeling of Multiple Jet V/STOL Aircraft Flow Fields in Ground Effect," presented at the Workshop on VSTOL Aerodynamics, Naval Postgraduate School, Monterey, Calif., May 1979.

[7] Kuhn, R.E., "An Empirical Method for Estimating Jet Induced Lift Losses of V/STOL Aircraft Hovering In and Out-of-Ground Effect," submitted to the Naval Air Development Center, Rept. R-AMPAC-105, Dec. 1978.

[8] Donaldson, C.D. and Snedeker, R.S., "A Study of Free Jet Impingement. Part I. Mean Properties of Free and Impinging Jets," *Journal of Fluid Mechanics*, Vol. 45, 1971, pp. 281-319.

[9] Kotansky, D.R., personal communication, Sept. 20, 1979.

[10] Schlichting, H., *Boundary Layer Theory*, 4th Ed., McGraw Hill Book Co., 1960, pp. 607-609.